

# XPS and NMR as Complementary Probes of Pseudogaps and Spin-charge Separation.

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**Abstract.** The possibility that strongly correlated many-electron systems may exhibit spin-charge separation has generated great excitement, particularly in the light of recent experiments on low dimensional conductors and high temperature superconductors. However, finding experimental support for this hypothesis has been made difficult by the fact that most commonly used probes couple simultaneously to spin *and* charge excitations. We argue that core hole photoemission (XPS)/nuclear magnetic resonance (NMR) couple independently and in exactly comparable ways to the local charge/spin susceptibilities of the system being measured. The explicit comparison of XPS and NMR data, particularly in systems which exhibit a pseudogap, may therefore yield fresh evidence for the existence (or non-existence) of spin-charge separation. Application of these ideas to the normal state of high temperature superconductors is discussed, and the application is further illustrated in some detail for quasi one-dimensional systems with charge density waves.

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## 1. Introduction

While the words “spinon” and “holon” are now firmly entrenched in the common vernacular of condensed matter physics, hard experimental evidence for the existence of low energy spin-charge separation in real materials has proved elusive. *Pseudogaps* — by which we mean behavior comparable to the opening of a gap but exhibited in the “normal” state of a system with an ordered groundstate — are on the other hand quite ubiquitous in those systems which are candidates for a spin-charge separated description. In this paper we will discuss what might be learned from the explicit comparison of core hole photoemission (XPS) and nuclear magnetic resonance (NMR) about two classes of material which display pseudogap behavior and may be spin-charge separated — the cuprate high temperature superconductors and quasi-one dimensional charge density wave systems.

The main reason for the difficulty in determining empirically whether a given system is spin-charge separated is that most experimental probes couple to both spin and charge excitations. In the case of angle resolved photoemission (ARPES), potentially one of the most powerful probes, a physical electron must be removed from the system under investigation, changing both its spin and charge quantum numbers. In order to extract information about spin-charge separation one must therefore have a concrete model giving predictions which are wholly dependent on the existence of spin-charge separation, against which the data may be checked. For this reason the search for spin-charge separation in low dimensional materials has often been equated with the search for hints of Luttinger Liquid (LL) behavior (noteably separate dispersing spin and charge peaks [1]) in ARPES spectra (see *e. g.* [2, 3, 4, 5]). Ingenious alternative approaches, such as the examination of spin-diffusion [6] in a 1D system, or the tunneling between 1D and 2D electron liquids [7], depend on exploiting a difference in the decay times or velocities for collective spin and charge excitations, and do not probe spin and charge separately. We note that evidence for Luttinger Liquid behaviour has been found recently from transport experiments on metallic Carbon Nanotubes [8].

More powerful general statements could be made about systems for which no detailed model existed if it were possible to directly compare two experiments which coupled independently to spin and charge excitations. We argue here that NMR, which has been used extensively in the investigation of pseudogap behavior in the cuprate high temperature superconductors [9], couples to spin excitations *in exactly the same way* that XPS couples to charge excitations, and that the two are therefore useful complementary probes of spin-charge separated behavior. We will also discuss how the existence of a pseudogap may in fact make it easier to distinguish spin-charge separated and non spin-charge separated theories.

We first briefly define spin-charge separation and review the relevant many body

effects which come into play in XPS and NMR. Since the goal of this paper is to motivate the comparison of two different experiments and not to provide a definitive treatment of any given set of results, we adopt a somewhat naive picture of both XPS and NMR. We hope in later papers to relax some of these simplifying assumptions.

## 2. Spin–charge separation

In atomic physics, quantum numbers may be classified by the symmetries to which they correspond, and broken up into groups according to the type of excitation (transition) considered. The assertion that a many body system is spin–charge separated is the claim that, at least at low energies, its energy levels may be classified by two *independent* sets of quantum numbers, one for spin and one for charge excitations. In terms of the Hamiltonian, this means

$$H = H_\rho + H_\sigma \quad [H_\rho, H_\sigma] = 0 \quad (1)$$

where  $H_\rho$  acts on the (Hilbert) space for charge and  $H_\sigma$  on the space for spin excitations. The condition (1) may not be strictly realized by microscopic model Hamiltonians relevant to experiment, but it remains a useful concept as long as coupling between spin and charge modes is sufficiently weak in the low energy limit.

As a consequence of (1) the state of the system factorizes

$$|\Psi_n\rangle = |\vec{\rho}\rangle \times |\vec{\sigma}\rangle \quad (2)$$

where  $n$  is the principal quantum number of the state and  $\vec{\rho}$  and  $\vec{\sigma}$  are (sets of) quantum numbers for spin and charge excitations.

Such a factorization would of course come as no surprise in the real–space world of atomic physics, but the independent electron model of metals (and band insulators) teaches us to think in terms of Bloch states — excitations of the many electron system are counted using the momentum quantum numbers of individual electrons, which means that quasiparticles *must* carry both the spin and charge of an electron. Interaction changes this picture very little within the usual Fermi liquid scenario, since its low lying excitations are by assumption electron–like quasiparticles with good momentum quantum numbers. Many different soluble models of interacting electrons in one dimension (for example the Tomonaga–Luttinger model discussed below) *do* however display spin–charge separation.

Spin–charge separation is then inherently a strong correlation effect — interaction between electrons should be important enough to render them completely incoherent, whilst at the same time stabilizing independent sets of quasi–particles with pure spin and charge quantum numbers (“spinons” and “holons”). In one dimension this occurs for arbitrarily weak interaction because charge confined to a line suffers rigorous phase

space constraints. In higher dimension interaction must be much stronger, or the phase space for electrons constrained by other considerations. We now turn to experiment.

### 3. XPS and NMR

#### 3.1. Experimental response functions.

NMR is an indirect probe of the spin susceptibility of an electron liquid. Excitations of the Nuclear lattice in a solid are damped by their interaction with the orbital and spin degrees of freedom of electrons. In a metal, the most important contribution to this damping comes from a hyperfine interaction with itinerant electrons and may be expressed in terms of a nuclear relaxation time  $T_1$  according to

$$\frac{1}{T_1 T} = \lim_{\omega \rightarrow 0} \sum_q F(q) \frac{\Im\{\chi_\sigma(q, \omega)\}}{\omega} \quad (3)$$

where  $T$  is the temperature of the system,  $\chi_\sigma(q, \omega)$  the dynamic spin-susceptibility of the electrons, and all relevant coupling constants have been absorbed into a weakly momentum dependent structure factor  $F(q)$ .

Provided that the momentum dependence of the structure factor can be neglected ( $F(q) \rightarrow F_0$ , equivalent to assuming a delta function interaction in real space),  $1/T_1 T$  is directly related to the imaginary part of the local spin-susceptibility. Evaluating this as single electron-hole bubble one then finds

$$\frac{1}{T_1 T} = F_0 n_0^2 \quad (4)$$

where  $n_0$  is the density of states at the Fermi surface. The result  $1/T_1 T = \text{const.}$  is known as the Korringa law.

Under certain plausible assumptions [10], XPS measures the spectral function  $A_c(\omega)$  of core electrons, which in most cases are highly localized and have very large binding energies. Core states in a metal are coupled by coulomb interaction with the itinerant charge of the conduction band. An infrared divergence in the number of charge carrying excitations of the itinerant electron liquid made by the photoemission of the core electron powerlaw divergence at threshold in their spectral function

$$A_c(\omega) \sim \frac{1}{(\omega - \epsilon_c)^{1-\alpha}} \quad (5)$$

characterized by an exponent  $\alpha$ . In the limit of weak coupling (essentially a second order linked cluster expansion [11]), we find

$$\alpha = \lim_{\omega \rightarrow 0} \sum_q |V_q|^2 \frac{\Im\{\chi_\rho(q, \omega)\}}{\omega} \quad (6)$$

and for an ordinary metal, under the assumption of local interaction ( $V_q \rightarrow V_0$ ),

$$\alpha = |V_0|^2 n_0^2 \quad (7)$$

which is once again independent of temperature.

Here we have deliberately adopted an idealized view of XPS line shapes, which are only strictly simple power laws at threshold and zero temperature, and in practice are considerably broadened by the (temperature independent) Auger decay of the core hole and a temperature dependent coupling to phonons. However none these mechanisms greatly affects the asymmetry of the line, which is still determined by  $\alpha$ , and good fits to experiment can usually be found by convoluting the simple power law Eq. (5) with Lorentzian (Auger process) and Gaussian (phonon process) lifetime envelopes to give the widely accepted Doniach–Sunjic lineshape [12].

### 3.2. Relevance to spin–charge separation

Within the picture developed above we see that XPS and NMR are *local zero frequency probes*, one coupling to charge and the other to spin excitations. Under the assumption of delta function interaction, both the Nuclear relaxation time  $1/T_1T$  and the core level asymmetry exponent  $\alpha$  are related to the local spin(charge) susceptibility *in exactly the same way*. They therefore form a complementary pair of probes which may be used to assess the degree to which spin and charge excitations are linked in any given system.

Without needing to have recourse to any specific model of the system under investigation, we can therefore directly compare the temperature dependence of  $\alpha$  and  $1/T_1T$ , either by plotting one against the other, or by examining the ratio

$$R(T) = \frac{\alpha}{1/T_1T} \quad (8)$$

as a function of temperature, rather as one might use the ratio of thermal and electrical conductivities to test the independent electron model of a metal (the Wiederman–Franz law). In this case or, in a Fermi liquid up to vertex corrections, one would find  $R = \alpha T_1T = \text{const.}$  This method of comparing temperature dependences becomes particularly powerful if one or both of the quantities becomes strongly temperature dependent, as would happen, for example, at the opening of a gap. This observation forms the basis of the remainder of our discussion of spin–charge separated systems.

## 4. Application to experiment

### 4.1. High temperature superconductors

Perhaps the most interesting candidates for a spin–charge separated description are the cuprate high temperature superconductors. Underdoped samples exhibit an exotic

metallic state with systematic unconventional temperature dependence of transport coefficients and, for some range of temperatures above the superconducting transition temperature  $T_c$ , evidence of a gap or “pseudogap” opening, at least for spin excitations. Spin-charge separated theories of the “normal” state of these systems have been advanced; in these the pseudogap which opens below some temperature  $T^*$  is uniquely associated with spin excitation — it is a spin-gap [13]. It is generally believed that below  $T_c$  spin and charge are reunited in Cooper pairs of d-wave symmetry with more or less free Fermionic quasiparticle excitations in the nodes of the gap. Alternative models of the pseudogap have been proposed which are not spin-charge separated. We contend that the comparison of XPS and NMR could help us to choose between them.

Important evidence for the opening of a pseudogap in the cuprates was gained from the temperature dependence of the local spin susceptibility, as measured by NMR [9]. If quasiparticles in the normal state are electron-like, as is generally the case in non spin-charge separated theories, then the pseudogap must reflect a general suppression of the density of states at the Fermi surface, and should be felt in the spin and charge channels. The temperature dependence of the local charge susceptibility, as measured by XPS, should therefore in the first approximation be the same as the temperature dependence of the local spin susceptibility. If, on the other hand, low energy spin and charge excitations “decouple” at  $T^*$ , this may have observable consequences for the evolution of NMR and XPS lineshapes in the pseudogap regime.

As stressed above, we have in the interests of clarity adopted a deliberately naive picture of the screening response which determines XPS lineshapes in strongly correlated electrons systems. In the case of the Cu-O lattice of the cuprate superconductors, a hybridization of metallic valence band states (Cu  $d$ -electrons) with ligand core levels (O  $p$ -orbitals) — the so called “Zhang-Rice singlet” [14] — is believed to be an essential part of the microscopic physics of these systems. The interplay of charge transfer and strong onsite interaction effects underlying this hybridization can have important consequences for photoemission, for example in the case of Cu  $2p$  states in the undoped cuprate  $\text{LaCuO}_3$ , where it leads to the appearance of a hierarchy of “satellite” peaks alongside the principle core line in XPS spectra [15]. Comparable interaction/charge transfer effects are believed to occur in many transition metal systems [16]. In fact this correlated screening of core holes on Cu site in cluster models of the Cu-O plane need not be local even in the sense of being restricted to reconfigurations of charge on the orbitals adjoining the “impurity” atom, but may extend across several unit cells [17]. Since photoemission is a surface sensitive technique, care must also be taken to eliminate spurious extrinsic features which mimic satellite structure from core spectra [18].

A complete theory of lineshapes in these systems would of necessity involve both a proper treatment of the many-orbital atomic physics of the Cu and O atoms, and the

many body physics of the conduction electrons, for each rival model of each phase of the system. None the less, the Doniach–Sunjic lineshape *has* proved useful in fitting some core levels in the cuprates, and many of the important low energy features of a successful many–body theory of the cuprates can be expected to be independent of the detailed structure of the underlying microscopic model. In particular the gross difference between low energy spin and charge susceptibilities should persist in any spin–charge separated scenario, and so the comparison of XPS and NMR should still be useful. The lineshapes of core levels of atoms neighbouring the  $Cu-O$  plane which are not hybridized strongly with the valence band electrons (for example the shallow  $p$  levels of  $La$  and  $Sr$  in  $(La_{1-x}Sr_x)_2CuO_{4-y}$  [19]) should in general be much simpler to calculate, although experimental spectra can be complicated by the near degeneracy of inequivalent levels on different atoms.

So far as we are aware, no serious analysis has been made of changes in core level lineshapes as a function of phase and temperature in the cuprates, even at the level of an attempt to extract the temperature dependence of the asymmetry exponent  $\alpha$ . However some evidence that the local spin susceptibility changes at  $T^*$  is provided by the shift of XPS lines at  $T^*$  in underdoped samples, which mimics that at  $T_c$  in overdoped samples [20]. A reduction of the local static charge susceptibility due to the opening of a gap will in general lead to a shift of lines to lower binding energies. Unfortunately this shift in XPS lines cannot be compared directly to the NMR Knight shift as the Knight shift is determined by the uniform and not the local spin susceptibility. None the less, as described above,  $1/T_1T$  and  $\alpha$  *can* be compared directly, and the comparison of their temperature dependences might yield interesting information about the relative effect of the pseudogap in spin and charge channels. The most interesting ranges of temperatures to study in this context would be the regions around  $T^*$  at which the pseudogap opens, and  $T_c$  at which spin and charge are implicitly believed to recombine.

#### 4.2. *Quasi–one dimensional CDW systems*

It is much easier to make concrete statements about spin–charge separation in quasi–one dimensional materials where the relevant theoretical models have already been widely studied. We therefore now turn to the analysis of XPS and NMR in a purely one–dimensional context, which we believe to be relevant to the opening of a pseudogap in quasi–one dimensional CDW systems. Our aim is to provide a useful starting point for comparison with experiment, not a definitive treatment of XPS in pseudogapped systems, but it is still important to restate the limitations of our perturbative approach.

The weak–coupling treatment of XPS spectra is based on the assumption that the lineshape is dominated by the powerlaw divergence with exponent  $1 - \alpha$  at threshold, and that  $\alpha$  is small enough to be reliably estimated from perturbation theory. Strictly

this is only true at vanishing core hole coupling and zero temperature, but lineshapes based on these assumptions have been applied successfully to metals over a wide range of temperatures, and so provide a reasonable starting point for any metallic system.

We assume that at sufficiently high temperatures the system is not only metallic but that  $\alpha$  is temperature independent, as in the free electron gas. We then calculate the dominant temperature corrections arising as the temperature of the system is reduced and the mechanism driving the pseudogap becomes effective. A number of special features arise in one-dimension (1D). These will be dealt with in context.

In a 1D free electron gas, the Fermi surface comprises distinct left and right Fermi points and there are two important scattering channels corresponding to “forward scattering” at either Fermi point (momentum transfer  $q \approx 0$ ), and “backward scattering” between Fermi points (momentum transfer  $q \approx \pm 2k_f$ ). There is no distinction between spin and charge susceptibilities, and in the weak coupling limit considered above the XPS exponent is given by

$$\alpha = n_0^2 \left[ |V_0|^2 + |V_{2k_f}|^2 \right] \quad (9)$$

and the NMR relaxation time by

$$\frac{1}{T_1 T} = n_0^2 \left[ F_0 + F_{2k_f} \right] \quad (10)$$

As remarked above, both  $\alpha$  and the  $1/T_1 T$  are *temperature independent* (the Korringa law). These expressions become temperature dependent when interaction is included in one of two ways. If a pseudogap opens through interaction with an “external field” (*i. e. without* destroying the electron-like nature of quasi-particles) it will *suppress* the density of states at the Fermi energy, giving the a temperature dependence to the prefactor  $n_0^2$  for both  $\alpha$  and  $1/T_1 T$ . In the case of one simple model of the pseudogap observed above  $T_{3D}$  in charge density wave (CDW) systems, the Lee, Rice and Anderson (LRA) model [21], the suppression of  $n_0^2$  with decreasing  $T$  may be calculated explicitly [22]. While the LRA model is somewhat crude, its application to the quasi-one dimensional CDW system  $(\text{TaSe}_4)_2\text{I}$  is successful enough that we consider it here as a valid phenomenological alternative to the strongly correlated “Luttinger Liquid” (LL) family of models [23].

Electron-electron interaction in one dimension, on the other hand, destroys all Fermionic quasiparticles and lends a temperature dependence to  $\alpha$  ( $1/T_1 T$ ) through the scaling of the  $2k_f$  “backscattering” component of the relevant charge (spin) susceptibility. This reflects the critical nature of the 1D interacting Fermi gas. The generic description of a one-dimensional Fermi liquid is the LL [24, 25], a fully spin-charge separated state (according to definition 1), whose excitations are long-wave length spin and charge density fluctuations described by the Tomonaga-Luttinger model

$$H^{LL} = H_\rho^{LL} + H_\sigma^{LL} \quad (11)$$

$$H_\rho = \frac{v_\rho}{2\pi} \int dx \left[ \frac{1}{K_\rho} (\partial_x \phi_\rho)^2 + K_\rho (\partial_x \theta_\rho)^2 \right] \quad (12)$$

$$H_\sigma = \frac{v_\sigma}{2\pi} \int dx \left[ \frac{1}{K_\sigma} (\partial_x \phi_\sigma)^2 + K_\sigma (\partial_x \theta_\sigma)^2 \right] \quad (13)$$

where  $v_{\rho(\sigma)}$  is the charge(spín) velocity, and  $K_{\rho(\sigma)}$  parameterizes interaction. In general, for repulsive interaction  $K_\rho < 1$  and spin-rotation invariance requires  $K_\sigma = 1$ . The bosonic field  $\phi_{\rho(\sigma)}$  and its canonical conjugate  $\partial_x \theta_{\rho(\sigma)}$  are related to the charge(spín) density and charge(spín) current density of physical electrons according to  $\rho_{\rho(\sigma)} = -\sqrt{2/\pi} \partial_x \phi_{\rho(\sigma)}$  and  $j_{\rho(\sigma)} = -\sqrt{2} K_{\rho(\sigma)} v_{\rho(\sigma)} \partial_x \theta_{\rho(\sigma)}$

One subtlety which must be kept in mind is that the independent spin and charge excitations for  $q \approx 0$  are mixed by processes involving a  $q \approx 2k_f$  momentum transfer. This is a special feature of one dimension which complicates but does not invalidate the type of analysis we wish to pursue.

For a Luttinger Liquid (Eqn. 11) the scaling of the XPS and NMR responses away from weak coupling may be described by

$$\alpha^{LL} = n_0^2 \left[ \beta_\rho |V_0|^2 + |V_{2k_f}|^2 \left( \frac{T}{T_0} \right)^{-\gamma} \right] \quad (14)$$

$$\frac{1}{T_1^{LLT}} = n_0^2 \left[ \beta_\sigma F_0 + F_{2k_f} \left( \frac{T}{T_0} \right)^{-\gamma} \right] \quad (15)$$

$$\beta_{\rho(\sigma)} = K_{\rho(\sigma)} \left( \frac{v_{\rho(\sigma)}}{v_f} \right)^2 \quad \gamma = 2 - K_\rho - K_\sigma \quad (16)$$

where  $T_0$  is a crossover temperature scale naively of order the bandwidth. In the non-interacting limit ( $K_\rho = K_\sigma = 1$ ,  $v_\rho = v_f$ ),  $\gamma = 0$  and  $\beta = 1$ , so we recover 9 and 10 [25]. In general  $\gamma > 0$  and  $2k_f$  contributions to the overall response are not suppressed but *enhanced* with decreasing temperature.

The results 14 and 15 are *only* valid as a description of scaling away from weak coupling; in the interest of simplicity we assume that is sufficient to consider only this regime. For spinless Fermions at  $T = 0$  it is believed that the backward scattering contribution to  $\alpha^{LL}$  takes on the universal value  $1/8$ ; in practise a transition to an ordered state at some intermediate temperature  $T_{3D}$  may prevent this strong coupling limit from ever being reached. Equation 15 represents the usual prediction for NMR in a LL and has been applied with some success to data taken on the quasi one-dimension Bechgaard salts [26]. Equation 14 is the equivalent prediction for XPS in a LL.

The difference between the temperature dependence of the local spin and charge susceptibilities of a LL is quite subtle because it is dominated by the mixing of spin and charge excitations through “backscattering” processes. However the LL does not offer a good description of the physics of quasi-one dimensional CDW systems such as  $(\text{TaSe}_4)_2\text{I}$ , as these exhibit clear evidence of a pseudogap and have dominant CDW

fluctuations.

We can break the symmetry between CDW and SDW fluctuations inherent in the LL and obtain a gap to spin excitations by introducing one further term in the Hamiltonian

$$H_\sigma^\Delta = \frac{2g_{1\perp}}{(2\pi\epsilon)^2} \int dx \cos(\sqrt{8}\phi_\sigma) \quad (17)$$

where  $g_{1\perp}$  is the strength of non current-conserving “backscattering” interactions and  $\epsilon$  a short distance cutoff. In the special case  $K_\sigma = 1/2$  the model may be solved explicitly by refermionization and it is found that the spin sector corresponds to a set of (massive) fermions with gap  $\Delta_\sigma = 2g_{1\perp}/2\pi\epsilon$ . The charge sector remains ungapped. It is known that the gap persists for  $K_\sigma \neq 1/2$ , although it no longer has this simple form. This new one dimensional electron liquid is known as the Luther Emery (LE) Liquid [27].

The opening of a spingap affects the temperature dependence of  $\alpha$  and  $1/T_1T$  in two ways. Firstly the contribution to the local spin susceptibility from long-wavelength spin fluctuations becomes activated, and is exponentially small at temperatures low compared with the spin-gap scale (measured pseudogaps in 1D systems are usually much larger than relevant experimental temperatures). Secondly, the existence of a spin coherence length  $\xi_\sigma^{-1} \sim \Delta_\sigma/v_\sigma$  cuts off critical scaling in the spin channel. The results (14) and (15) are therefore modified to

$$\alpha^{LE} = n_0^2 \left[ \beta_\rho |V_0|^2 + |V_{2k_f}|^2 \left( \frac{T}{T_0} \right)^{-\gamma} \right] \quad (18)$$

$$\frac{1}{T_1^{LE}T} = n_0^2 \left[ \beta_\sigma F_0 e^{-\Delta_\sigma/T} + F_{2k_f} \left( \frac{\Delta_\sigma}{T_0} \right)^{-\gamma} \right] \quad (19)$$

where  $\beta_\sigma \sim O(1)$  and  $\Delta_\sigma \sim v_f/\epsilon \exp(\pi v_f/g_1)$  are for  $K_\sigma \neq 1/2$  undetermined coefficients.

The independence of spin and charge excitations is now manifest in the very different temperature dependences of  $\alpha$  and  $1/T_1T$ . The ratio  $R(T)$ , our proposed diagnostic for spin charge separation, is shown for each case in Table 4.2, where the coefficients  $a, b, c$  *etc.* as well as the exponent  $\gamma$  and spingap  $\Delta_\sigma$  are to be determined empirically. The result for the LE liquid is clearly the “smoking gun” — it is very hard to conceive any non spin-charge separated scenario in which the local spin-susceptibility could be activated while the local charge was not.

One could repeat these arguments for a term like Equation (17) in the charge sector (Umklapp term, relevant at commensurate filling and in Mott insulators), with the understanding that in this case a gap opens in the charge and not the spin sector and so it is  $\alpha$ , not  $1/T_1T$ , which displays activated behavior.

**Table 1.** Ratio  $R(T)$  for various one-dimensional electron liquids.

	$R(T)$
Free $e^-$	const.
LRA	const.
LL	$(a + bT^{-\gamma})/(c + dT^{-\gamma})$
LE	$(a' + b'T^{-\gamma})/(c' + d'e^{-\Delta_\sigma/T})$

## 5. Conclusions

XPS and NMR, as probes of local charge and spin susceptibility, are individually very informative about the opening of pseudogaps in strongly correlated systems. The direct comparison of the two offers the possibility of determining whether a gap has opened to spin excitations without a gap opening to charge excitations (or vis-versa). Since the opening of a gap to spin(charge) excitations without an accompanying gap to charge(spin) excitations requires spin-charge separation in the underlying physics, together they may be used to probe directly for spin-charge separation. It is anticipated that many of the arguments presented above with relation to the LE Liquid may be carried over directly to assessment of the opening of a “spin-gap” in the pseudogap phase of the underdoped cuprates.

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